Binary Options

QF 301

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Binary Options

Two simple types :

1. Cash or Nothing
2. Asset or Nothing

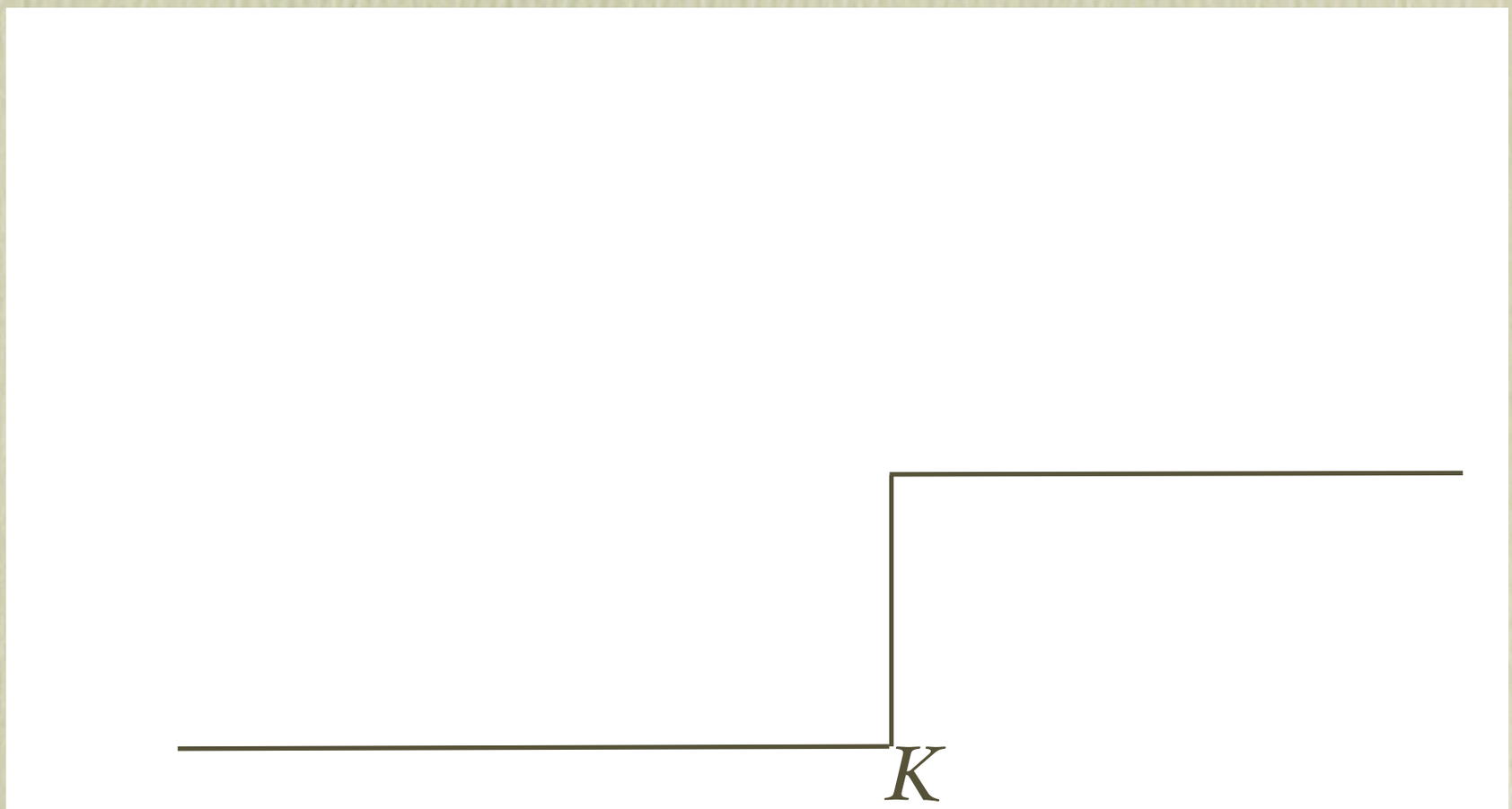
If the asset price at maturity is higher than the strike, payoff is **$1** (or some other pre-determined amount) else it is zero.

This is also called a 'bet'

This is simple to price because it is the probability of receiving **$1** at maturity, discounted to today.

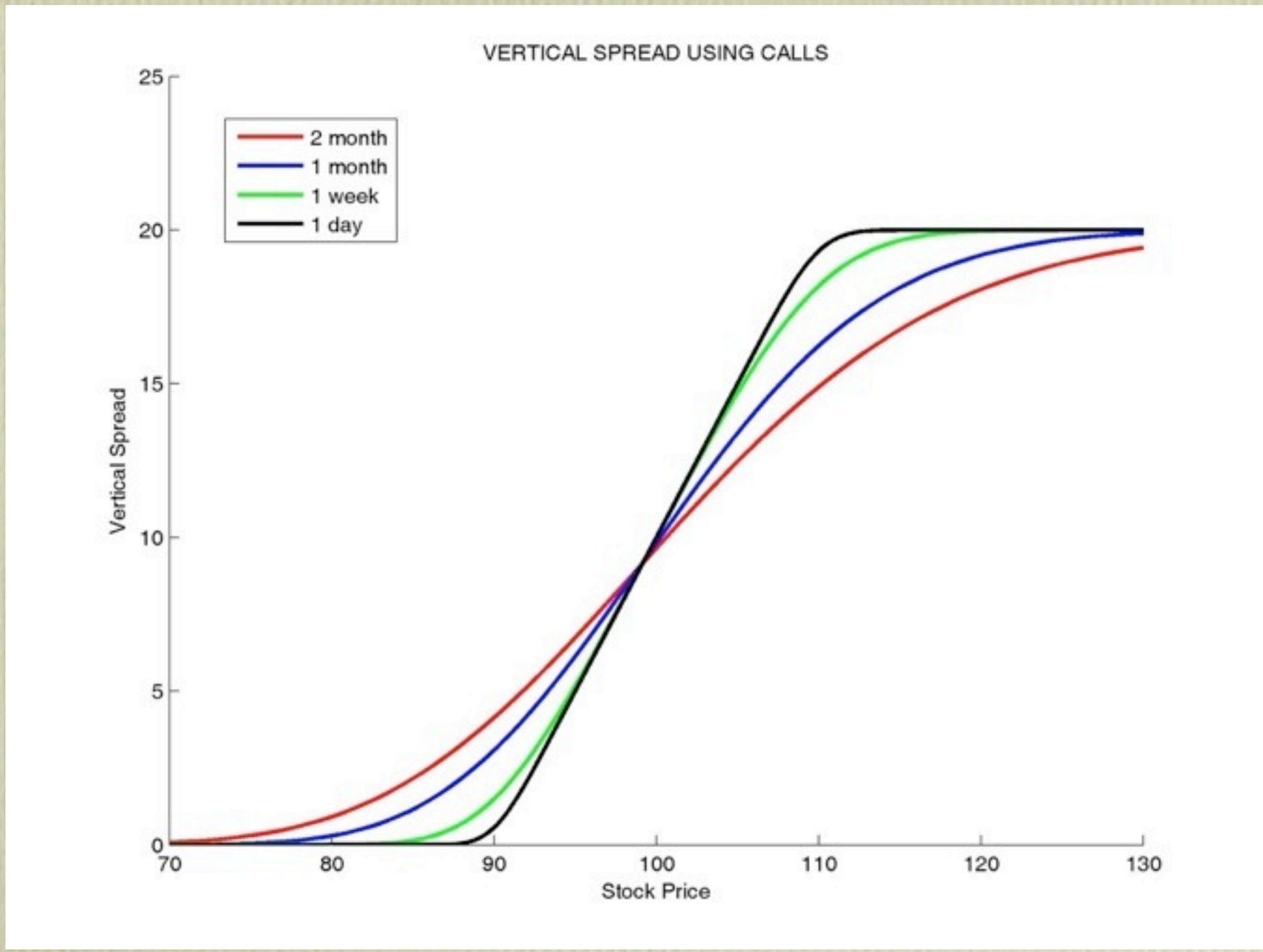
$e^{-rT}N(d_2)$

Cash or Nothing Call

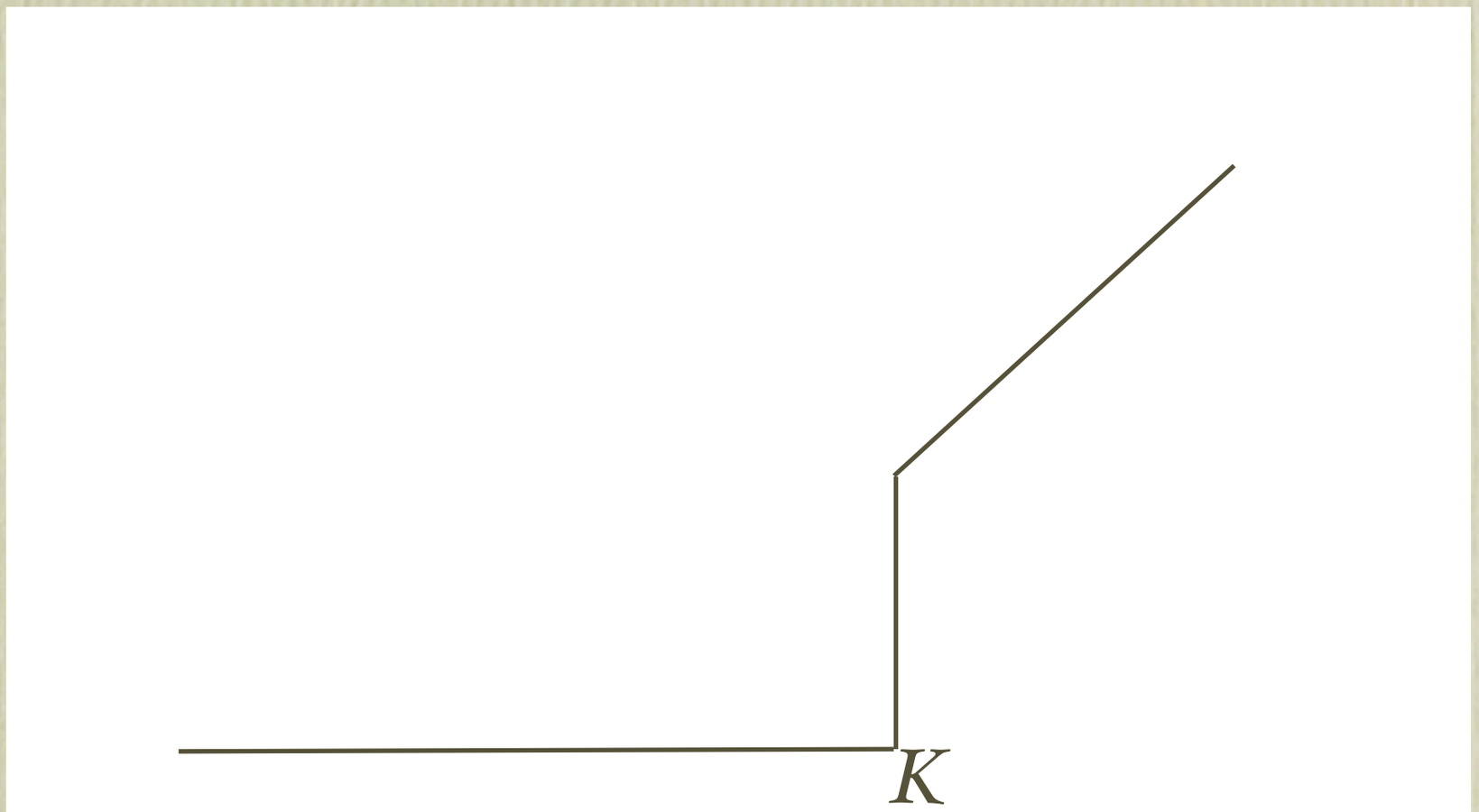


The payoff is \$1 , if S > strike K at maturity; 0 otherwise

Constructing a Binary Call from a Vertical Call Spread

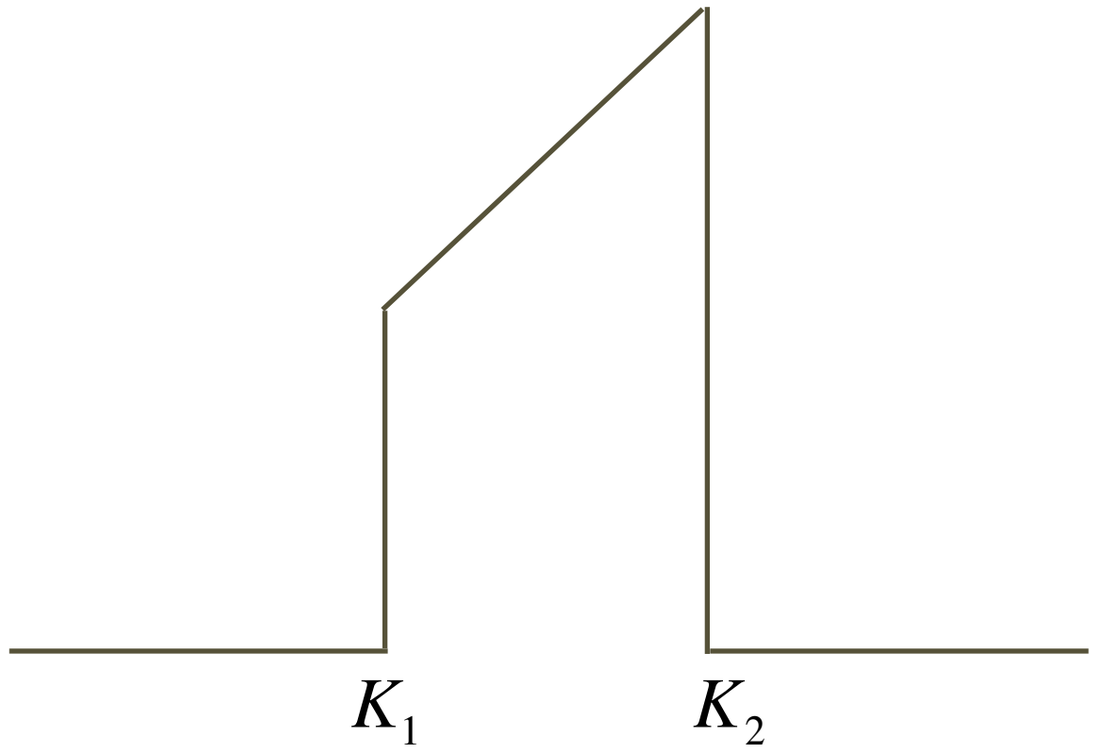


Asset or Nothing Call



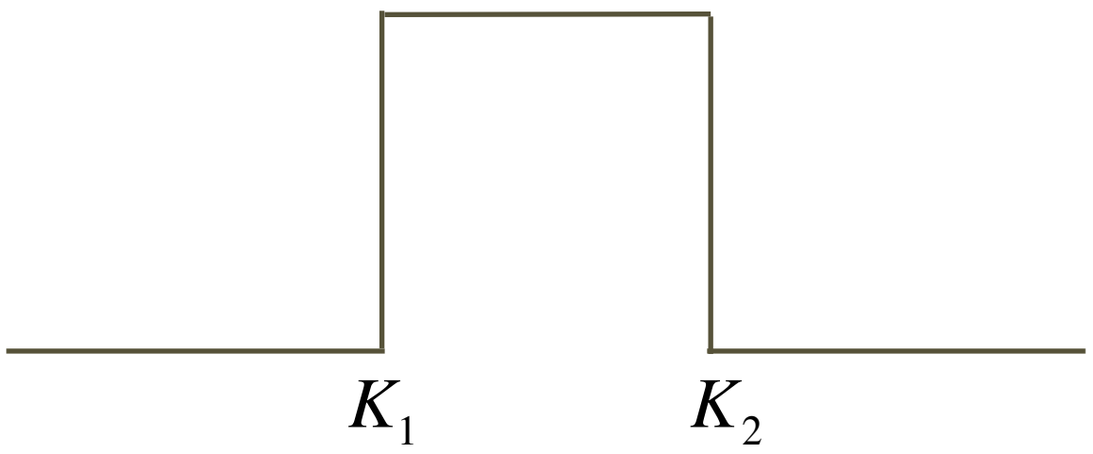
The payoff is one share, if $S >$ strike K at maturity; 0 otherwise

Super Share



The payoff is one share, if S is between K_1 and K_2 at maturity; 0 otherwise

Step Structure



The payoff is \$1 , if S is between K_1 and K_2 at maturity; 0 otherwise

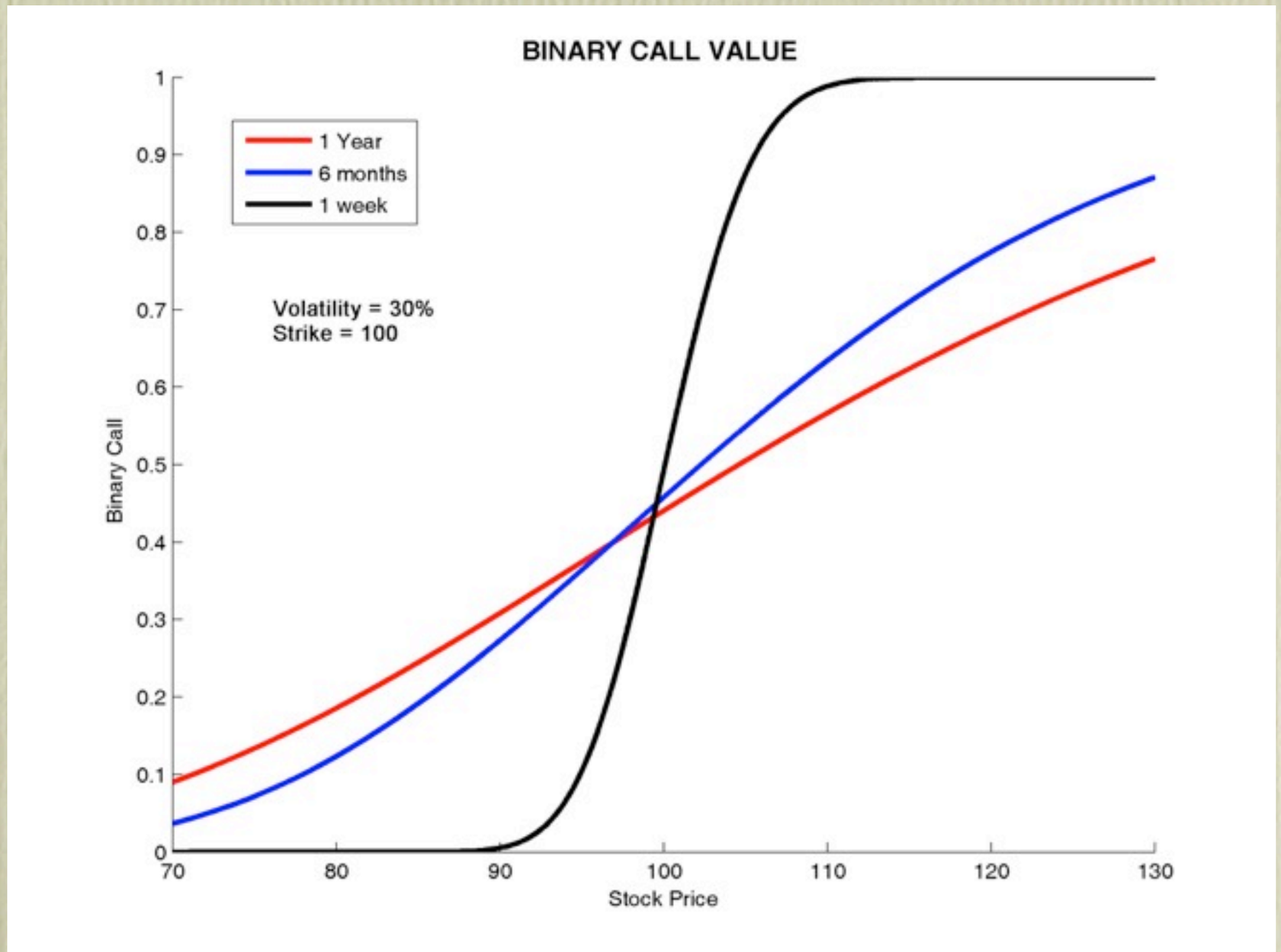
Range Accrual Notes

- A Range Accrual note will pay higher than market coupon for each day the reference asset is within a certain range.
- For example, for each day in the first 6 months that LIBOR stays between 2% and 4%, the coupon might accrue at LIBOR+125 bps; and zero otherwise.
- Risk managed by a portfolio of digital options expiring at each date struck at lower and upper limits

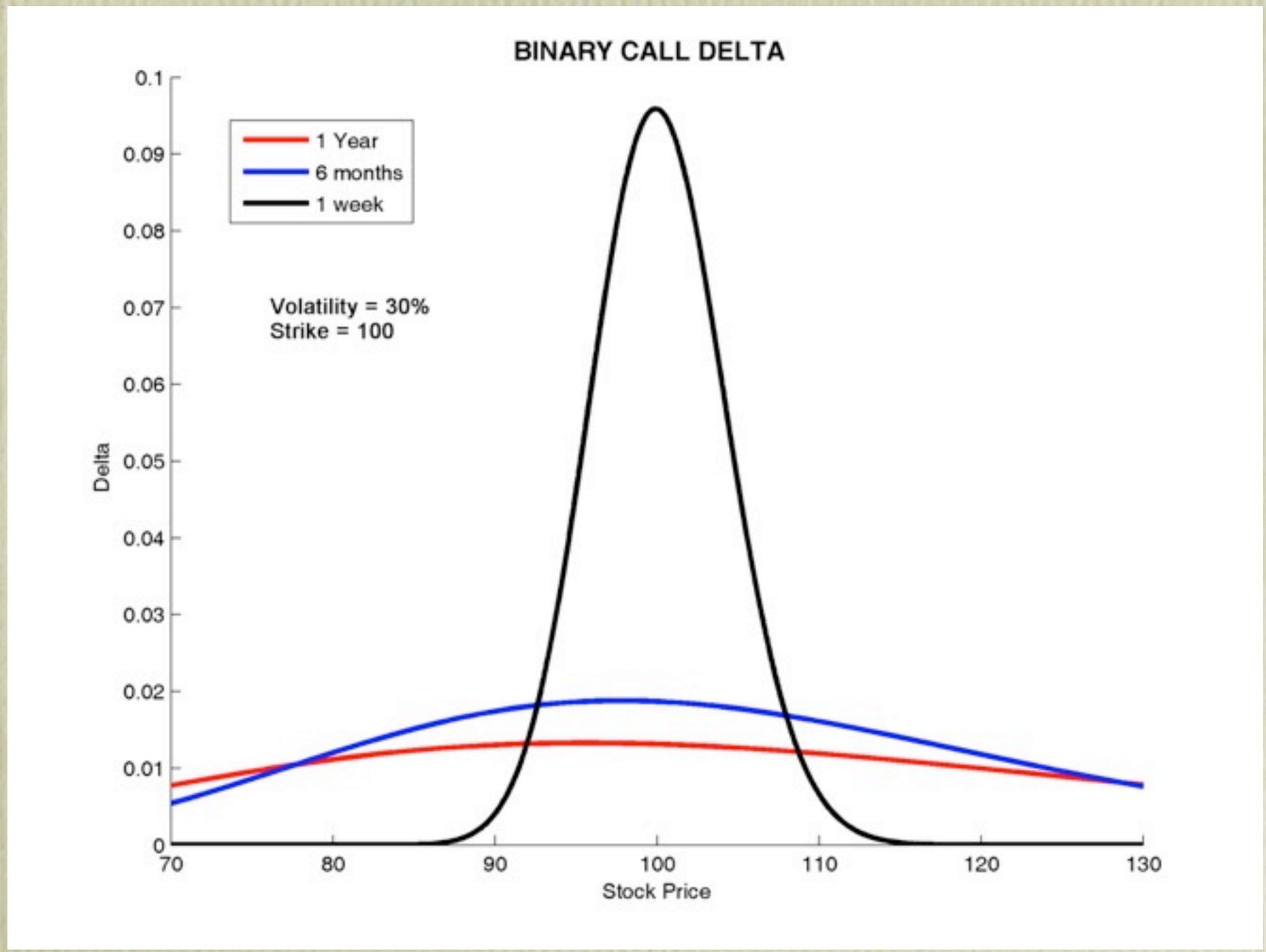
Binary Option

VALUE	Calls	Puts
Option Price	$e^{-r\tau}\Phi(d_2)$	$e^{-r\tau}\left(1-\Phi(d_2)\right)$
Delta	$\frac{e^{-r\tau}\varphi(d_2)}{\sigma S\sqrt{\tau}}$	$-\frac{e^{-r\tau}\varphi(d_2)}{\sigma S\sqrt{\tau}}$
Vega	$-e^{-r\tau}\varphi(d_2)\frac{d_1}{\sigma}$	
Theta	$re^{-r\tau}\Phi(d_2)+e^{-r\tau}\varphi(d_2)\times\left(\frac{d_1}{2\tau}-\frac{r-q}{\sigma\sqrt{\tau}}\right)$	$re^{-r\tau}\left(1-\Phi(d_2)\right)-e^{-r\tau}\varphi(d_2)\times\left(\frac{d_1}{2\tau}-\frac{r-q}{\sigma\sqrt{\tau}}\right)$
Gamma	$-e^{-r\tau}\frac{d_1\varphi(d_2)}{\sigma^2S^2\tau}$	

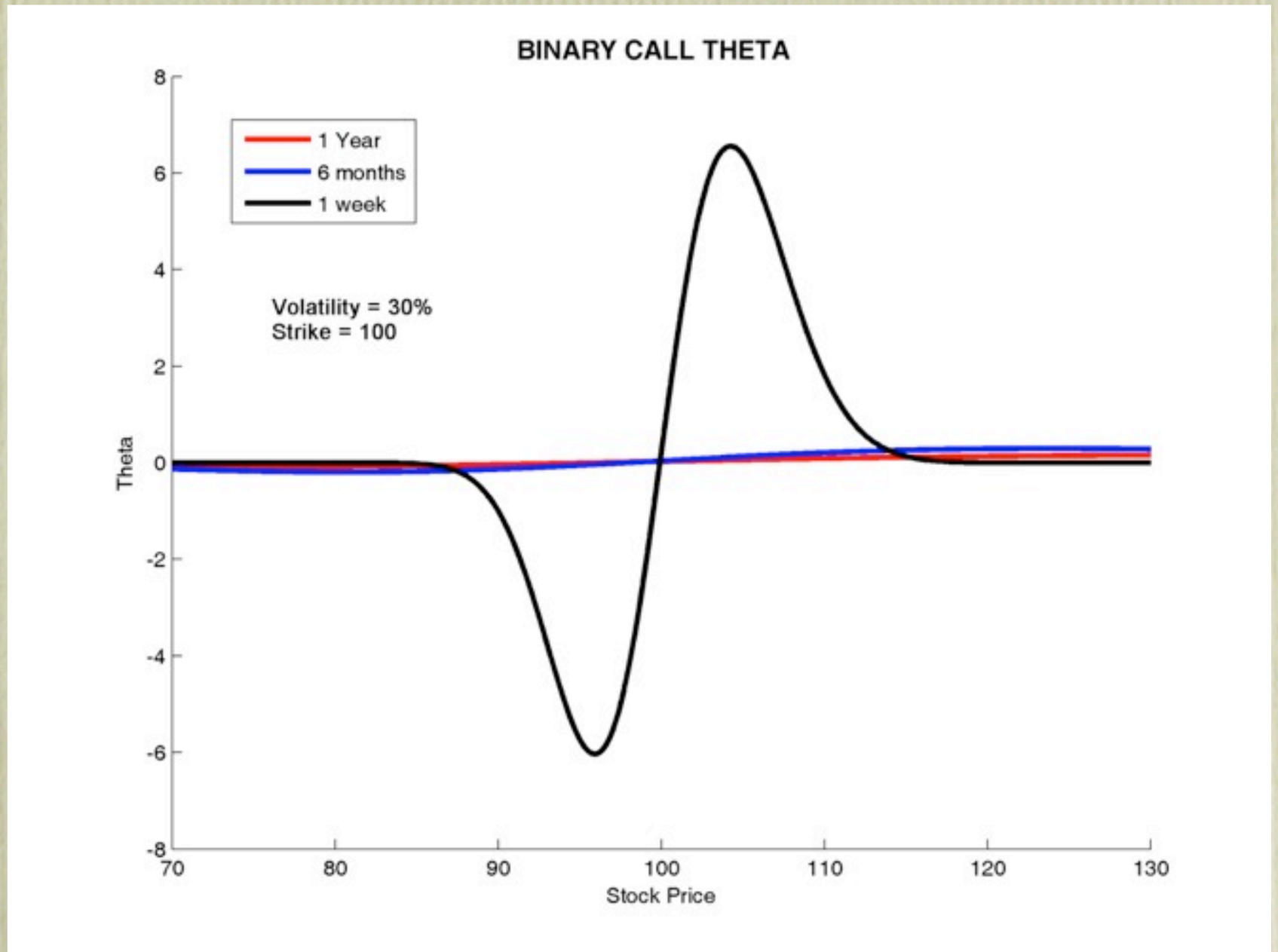
Binary Call: Value



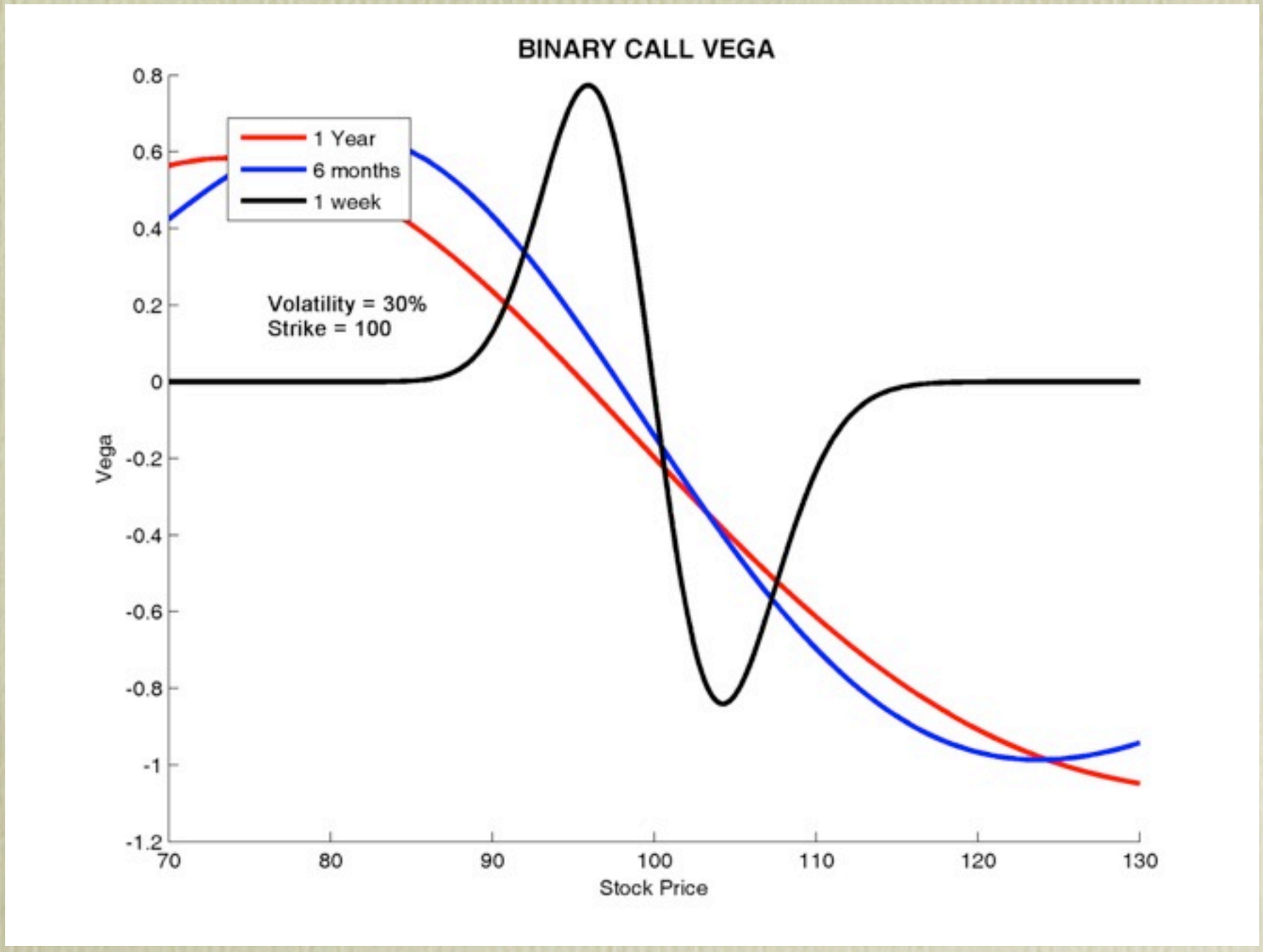
Binary Call: Delta



Binary Call: Theta



Binary Call: Vega



Runs and Randomness

- A run within a series is a sequence of adjacent elements with same value: HHHTHHTT has a run of 3 heads, a run of 1 T, then a run of 2 H and finally, a run of 2 T.

- If you toss a fair coin 100 times: Q1. how many runs would you expect to observe? Q2.What do you think might be the length of the longest run of heads? Q3.The length of longest run of tails? Q4.The length of the longest run?

- On being asked to imagine tossing a fair coin and writing down the answers, most people have shorter “longest runs” than real coin toss sequences.This fact is often used to separate “real” random sequences from “fake” random ones.

How to Check If a Coin is Fair?

- Bayes' theorem states that the *posterior* distribution is proportional to (*prior * likelihood*)

- For a Bernoulli RV with unknown probability of success *p* in [0,1], a commonly used conjugate prior is the Beta distribution. A Beta distribution has two parameters, α and β. For α =*1*, β=*1*, the Beta distribution is just the Uniform distribution on [0,1].

- If a head is regarded as a success, on N coin tosses, if there are *s* heads and *f (=N-s)* tails, the posterior distribution is another Beta distribution :
Beta (α +*s*, β +*f*). This posterior can be the new prior.

Simulating Randomness Can Be Tough: Weldon's Dice Data

- Walter F.R. Weldon (1860-1906), an English biologist tossed 12 dice 26,306 times and noticed that 5 and 6 occurred more often than theory would suggest.

- A Chi-square test was conducted to check if the large deviations observed could be just due to randomness. A highly impressive *p*-value of 0.000016 forcefully suggested that the dice showed a bias towards 5 and 6.

- Karl Pearson estimated this was due to poorly constructed dice - cheap dice have hollowed out pips, and thus the face 6, having six hollows, is lighter than its opposite face, which has 1 hollow or pip. (Opposite faces sum to 7).

Coins and the Father of Game Theory

- How thick should a fair coin be, so that the probability of landing on the edge, landing heads,landing tails are all equal? Someone asked this of John von Neumann who not only solved the problem in about 20 seconds, but also provided the answer to three places of decimals!

- A biased coin can be used to simulate the tossing of a fair coin - always toss twice; if both times the same face shows up ignore & toss twice again. However, if one toss shows head and the other tail, always select what shows on the first toss and you have simulated a fair coin. Always selecting the second toss is equivalent. Algorithm suggested by the inimitable von Neumann

A Magical Interlude: Persi Diaconis

- Stanford Mathematician and Magician
- Left home at 14 to travel with the famous sleight of hand artiste Dai Vernon, dropped out of high school.
- At 24, returned to night classes at college; magic tricks during day to pay for it.
- Martin Gardener included two of his card tricks in his list of 10 greatest ever card tricks.
- If you hit a coin with the same force in the same place, it always does the same thing," he says.

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Product Specifications

CBOE BINARY OPTIONS ON THE S&P 500® INDEX (SPX℠)

Description:
CBOE Binary Options are contracts that have an "all-or-nothing" payout depending on the settlement price of the underlying broad-based index relative to the strike price of the binary option.

Binary Call Options pay either 1) a fixed cash settlement amount, if the underlying index settles ***at or above*** the strike price at expiration; or 2) nothing at all, if the underlying index settles below the strike price at expiration. Binary Put Options pay either 1) a fixed cash settlement amount, if the underlying index settles ***below*** the strike price at expiration; or 2) nothing at all, if the underlying index settles at or above the strike price at expiration.

Underlying:
S&P 500 Index (SPX)

Symbols:
BSZ

Multiplier:
$100

Strike Price Intervals:
Strike prices may be listed with a minimum interval of 5 points.

Comparison of Binary Call with Standard Call

Strike = 100

T = 0.5

Volatility = 25%

r = 0

q = 0

S	80	90	100	110	120
VanillaCall	0.78	2.84	7.04	13.44	21.52
BinaryCall	0.09	0.25	0.46	0.67	0.83